Calc Q2 Test 3 Review Key

1) $f(x) = 4x^3 - x^4$

$f'(x) = 12x^2 - 4x^3$

$0 = 12x^2 - 4x^3$

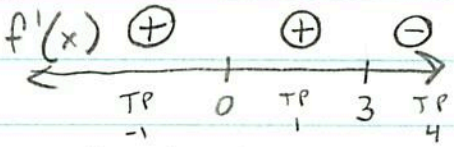
$0 = 4x^2(3 - x)$

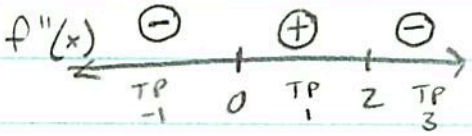
$x = 0 \mid x = 3$

$f''(x) = 24x - 12x^2$

$0 = 12x(2 - x)$

$x = 0 \mid x = 2$





a) $(-\infty, 0) \cup (0, 3)$

b) $(3, \infty)$

c) $(0, 2)$

d) $(-\infty, 0) \cup (2, \infty)$

e) Inflection Pts

$(0, 0)$ and $(2, 16)$

$f(0) = 4(0)^3 - (0)^4$ $\quad f(2) = 4(2)^3 - 2^4$

2) $f(x) = \dfrac{2x}{x-2}$

$f'(x) = \dfrac{(x-2)(2) - (2x)(1)}{(x-2)^2}$

$f'(x) = \dfrac{2x - 4 - 2x}{(x-2)^2}$

$f'(x) = \dfrac{-4}{(x-2)^2}$

$f'(x) = 0$ Never

$f'(x) = \text{und}$ $\quad x - 2 = 0$ $\quad x = 2$

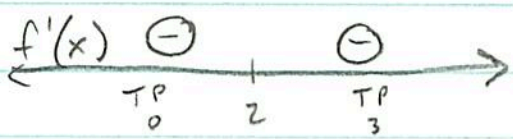
$f''(x) = \dfrac{(x-2)^2(0) - (-4)(2(x-2)(1))}{(x-2)^4}$

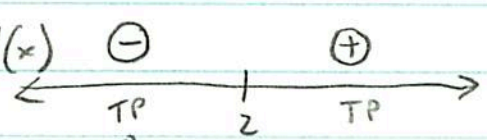
$f''(x) = \dfrac{0 + 8(x-2)}{(x-2)^4}$

$f''(x) = \dfrac{8(x-2)}{(x-2)^4} = \dfrac{8}{(x-2)^3}$

$f''(x) = 0$ Never

$f''(x) = \text{und.}$ $\quad x - 2 = 0$ $\quad x = 2$





a) Never

b) $(-\infty, 2) \cup (2, \infty)$

c) $(2, \infty)$

d) $(-\infty, 2)$

(No inflection pt) e) Inflection pt at $x = 2$

But function DNE there

3) $f(x) = (2x-5)^3$

$f'(x) = 3(2x-5)^2(2)$

$f'(x) = 6(2x-5)^2$

$0 = 6(2x-5)^2$

$0 = 2x-5$

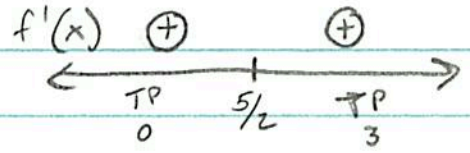
$\frac{5}{2} = x$

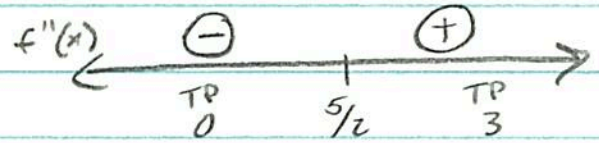
$f''(x) = 12(2x-5)(2)$

$f''(x) = 24(2x-5)$

$0 = 24(2x-5)$

$x = \frac{5}{2}$





a) $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

b) Never

c) $(\frac{5}{2}, \infty)$

d) $(-\infty, \frac{5}{2})$

e) Inflection Pt @ $x = \frac{5}{2}$

$f(\frac{5}{2}) = (2(\frac{5}{2})-5)^3$

$f(\frac{5}{2}) = 0$

$(\frac{5}{2}, 0)$

4) $f(x) = \frac{x^2}{x^2-4}$

$f'(x) = \frac{(x^2-4)(2x) - (x^2)(2x)}{(x^2-4)^2}$

$f'(x) = \frac{2x^3 - 8x - 2x^3}{(x^2-4)^2}$

$f'(x) = \frac{-8x}{(x^2-4)^2}$

$f''(x) = \frac{(x^2-4)^2(-8) - (-8x)(2)(x^2-4)(2x)}{(x^2-4)^4}$

$f''(x) = \frac{-8(x^2-4)^2 + 32x^2(x^2-4)}{(x^2-4)^4}$

$f''(x) = \frac{-8(x^2-4) + 32x^2}{(x^2-4)^3}$

$f''(x) = \frac{-8x^2+32+32x^2}{(x^2-4)^3} = \frac{24x^2+32}{(x^2-4)^3}$

$f'(x) = 0$

$-8x = 0$

$x = 0$

$f'(x) = \text{und}$

$x^2-4 = 0$

$x = \pm 2$

$f''(x) = 0$

$24x^2 + 32 = 0$

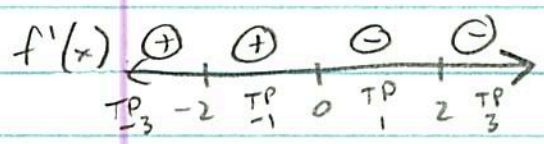
$8(3x^2+4) = 0$

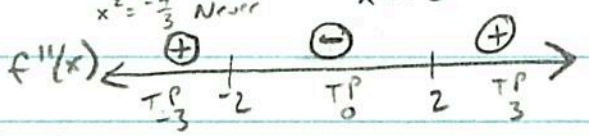
$x^2 = -\frac{4}{3}$ Never

$f''(x) = \text{und}$

$x^2-4 = 0$

$x = \pm 2$





a) $(-\infty, -2) \cup (-2, 0)$

b) $(0, 2) \cup (2, \infty)$

c) $(-\infty, -2) \cup (2, \infty)$

d) $(-2, 2)$

e) None

5) $f(x)$ is increasing when $f'(x)$ is positive (above the x-axis)

$(-1, 3) \cup (5, \infty)$

6) $f(x)$ is decreasing when $f'(x)$ is negative (below the x-axis)

$(-\infty, -1) \cup (3, 5)$

b) $f(x)$ is concave up when $f'(x)$ is increasing

$(-\infty, \frac{1}{2}) \cup (4, \infty)$

$f(x)$ is concave down when $f'(x)$ is decreasing

$(\frac{1}{2}, 4)$

c) $f(x)$ has an inflection pt when $f'(x)$ has a max or min

Inflection pt when $x = \frac{1}{2}$ and $x = 4$

7) I → C

II → B

III → A

8)a) $f(x) = 2x^3 - 9x^2 + 12x - 5$ $[0, 2]$

$f'(x) = 6x^2 - 18x + 12$

$0 = 6(x^2 - 3x + 2)$

$(x-2)(x-1)$

$x = 2 \quad x = 1$

$f(0) = 2(0)^3 - 9(0)^2 + 12(0) - 5 = \boxed{-5}$ Abs Min

$f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 5 = \boxed{0}$ Abs Max

$f(2) = 2(2)^3 - 9(2)^2 + 12(2) - 5 = -1$

b) $f(x) = 2x^3 - 15x^2 + 24x + 2$ $[0, 2]$

$f'(x) = 6x^2 - 30x + 24$

$6(x^2 - 5x + 4)$

$(x-4)(x-1)$

$x = 4 \quad x = 1$

Not in interval

$f(0) = 2(0)^3 - 15(0)^2 + 24(0) + 2 = \boxed{2}$ Abs Min

$f(1) = 2(1)^3 - 15(1)^2 + 24(1) + 2 = \boxed{13}$ Abs Max

$f(2) = 2(2)^3 - 15(2)^2 + 24(2) + 2 = 6$

9) a) $f'(-2) < 0$

b) $f'(-1) > 0$

c) $f'(0) = 0$

d) $f(1.5) > 0$

e) $f'(-3) > f'(-1)$ Tangent line @ $f'(-3)$ is steeper than @ $f'(-1)$

f) $f(1) > f'(1)$ $f(1) = 0$

g) $f''(1) > f'(1)$ $f(x)$ is concave up at $x=1$ therefore $f''(x)$ is positive

h) $f'(1) < f'(0)$